Answers for Homework Handout for unit 4

Work and Energy

1. Here we use $W = \vec{F} \odot \vec{D} = FD\cos\theta$:

   a) $W = (30\,\mathrm{N})(12\,\mathrm{m})\cos 0° = 360\,\mathrm{J}$.

   b) $W = (710\,\mathrm{N})(3.2\,\mathrm{m})\cos 31° = 1.95 \times 10^3\,\mathrm{J}$.

   c) $W = (55\,\mathrm{N})(10\,\mathrm{m})\cos 180° = -5.5 \times 10^2\,\mathrm{J}$.

2. One key phrase is "constant velocity," meaning that $F_{\mathrm{net}} = 0$ and thus $W_{\mathrm{net}} = 0$. (The work-energy theorem gives the same result, because $\Delta K = 0$, also.) That answers part (b). For part (a), we find the applied force F_{app} that is balanced by the kinetic frictional force, $F_f = \mu_k F_N$. From Newton's second law, we determine that $F_N = F_g = mg = 833\,\mathrm{N}$. So the work done by the pusher is 1.76×10^3 J.

3. Here we assume that the patient starts at rest and finishes at rest, so that $W_{\mathrm{net}} = 0$ by the work-energy theorem. The only forces doing work are gravity (F_g) and the applied force (F_{app}). The work done by gravity depends only on the change in height, $W_g = -mg(\Delta y)$, so W_{app} must be equal and opposite since the net work is zero: $W_{\mathrm{app}} = mg(\Delta y) = 330\,\mathrm{J}$.

4. The spring constant comes from $F_{app} = k(\Delta x)$; $k = (200\,\mathrm{N})/(0.025\,\mathrm{m}) = 8000\mathrm{N/m}$ Then the work done $W = \frac{1}{2}k(\Delta x)^2 = 7.1\,\mathrm{J}$.

5. battery: chemical/electrical energy

   sling shot: elastic (spring) potential energy

   wood and oxygen: chemical

   climber on mountain: gravitational potential energy

   spinning top: (rotational) kinetic energy

   pot of hot water: thermal (kinetic) energy

   x-rays: electromagnetic energy (not really "storage" per se, however)

6. From the top to the bottom, the gymnast's center of mass moves downward by 2 m ($\Delta y = -2\,\mathrm{m}$), allowing gravity to do positive work (i.e., turning gravitational potential energy into kinetic energy). We assume that the friction of the hands on the bar does negligible work and can be ignored. The tension force in the arms also does zero work because it is always perpendicular to the direction of motion. By the work-energy theorem,

   $$W_{\mathrm{net}} = K_f - K_i$$

   the only force doing work is gravity, and $K = \frac{1}{2}mv^2$, so

   $$W_g = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

   and solve for v_f after substituting $W_g = -mg(\Delta y)$. Note that the mass divides out of the equation! ($v_f = 6.37\,\mathrm{m/s}$)

7. In this problem we can use the work-energy theorem to find the answer to (b) first and use that answer to find (a). In class you also saw how to do part (a) by using forces. The only acting forces are gravity and the normal force (F_N), and F_N does zero work because it is perpendicular to the displacement ($\cos 90° = 0$). Work-energy tells us that $W_{\rm net} = K_f - K_i$, and we know that $K_f = 0$ (comes to rest at the top of the incline). The only work comes from $W_g = -mg(\Delta y)$, where Δy comes from the geometry of the incline: $\sin 10° = \frac{(\Delta y)}{L}$ (L = length of incline = 0.80 m). So $\Delta y = 0.138m$ and $K_i = 1.01$ J. Then for part (a), $v_i = 1.6\, m/s$ (from $K_i = \frac{1}{2}mv_i^2$).

8. Another use for the work-energy theorem! We know the mass of the block and its initial and final speeds, so we know $K_i = 0$ and $K_f = \frac{1}{2}mv_f^2$. The work done by friction is $W_f = F_f D \cos 180°$ (where $D = 0.9$ m and $F_f = \mu F_N = \mu F_g = \mu mg$ by Newton's law and $\mu = 0.4$). The spring work is $W_s = \frac{1}{2}k(\Delta x)^2$, where $\Delta x = 0.20$ m.

$$W_{\rm net} = K_f - K_i$$

$$W_f + W_s = \frac{1}{2}mv_f^2 - 0$$

$$-\mu mgD + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_f^2$$

Plugging in numbers and solving for the spring constant, we get $k = 8150$ N/m.

Power and Efficiency

9. The power (1200 W) is the work done per time: $P = W/(\Delta t)$. So we must find the work done to calculate the time. The work done by the motor is the work against gravity, so $W_m = -W_g = mg(\Delta y) = 1.66 \times 10^4$ J, so $\Delta t = 13.7$ s. [The only forces doing work are the applied force and gravity, and the net work is zero (constant velocity).]

10. Efficiency is $\varepsilon = W_{\rm out}/W_{\rm in}$. Here, $W_{\rm out} = 72$ J and $W_{\rm in} = 600$ J, so $\varepsilon = 0.12$ (or 12%).

11. $W_{\rm in} = 48000$ J and $W_{\rm out} = W_{\rm in} - W_{\rm waste} = 15000$ J. So $\varepsilon = 0.31$ (or 31%).

Momentum and Impulse

12. a) The momentum gained by the wall is the opposite of the change in momentum of the ball (by conservation of momentum). $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. If we take the initial direction of the ball to be the positive $\hat{x}$ direction, then $\vec{p}_i = m\vec{v}_i = (0.5\,{\rm kg})(12\,{\rm m/s}\,\hat{x}) = 6\,{\rm kg\,m/s}\,\hat{x}$, and $\vec{p}_f = -6\,{\rm kg\,m/s}\,\hat{x}$. So $\Delta\vec{p}_{ball} = -12\,{\rm kg\,m/s}\,\hat{x}$ and $\Delta\vec{p}_{wall} = -\Delta\vec{p}_{ball} = 12\,{\rm kg\,m/s}\,\hat{x}$.

    b) The impulse is $\vec{F}_{\rm ave}(\Delta t) = \Delta\vec{p}$. So for $\Delta t = 0.01\,$s, $\vec{F}_{\rm ave} = 1200\,$N.

13. Momentum is conserved because the net external force is zero, so that $\vec{p}_{tot,f} = \vec{p}_{tot,i}$. The astronaut and wrench are initially "at rest" (with respect to the spacecraft), so $\vec{p}_{tot,i} = 0$ ($v_{a,i} = 0$ and $v_{w,i} = 0$). So we have for the final total momentum (using $\vec{v}_{w,f} = 5\,{\rm m/s}\,\hat{x}$):

$$\vec{p}_{tot,f} = 0$$

$$m_a\vec{v}_{a,f} + m_w\vec{v}_{w,f} = 0$$

$$m_a\vec{v}_{a,f} = -m_w\vec{v}_{w,f}$$

$$\vec{v}_{a,f} = \frac{-m_w\vec{v}_{w,f}}{m_a} = -0.083\,\mathrm{m/s}$$

(The negative sign indicates the opposite direction from the wrench.)

14. Again, $\vec{p}_{tot,f} = \vec{p}_{tot,i}$ by momentum conservation (zero net external force on the system of the student [st] and snowball [sb]). In this case $\vec{v}_{\mathrm{st,i}} = 0$ and $\vec{v}_{\mathrm{sb,i}} = 22\,\mathrm{m/s}\,\hat{x}$. For the final momentum, the two masses stick together and move with a common velocity $\vec{v}_f$:

$$\vec{p}_{tot,f} = \vec{p}_{tot,i}$$

$$m_{tot}\vec{v}_f = m_{st}\vec{v}_{st,i} + m_{sb}\vec{v}_{sb,i}$$

$$(m_{st} + m_{sb})\vec{v}_f = 0 + m_{sb}\vec{v}_{sb,i}$$

Then $\vec{v}_f = 0.13\,$m/s.

15. a) Same direction:

$$\vec{p}_{tot,f} = \vec{p}_{tot,i}$$

$$m_{tot}\vec{v}_f = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$(60,000\,\mathrm{kg})\vec{v}_f = (40,000\,\mathrm{kg})(3.0\,\mathrm{m/s}\,\hat{x}) + (20,000\,\mathrm{kg})(5.0\,\mathrm{m/s}\,\hat{x})$$

Then $\vec{v}_f = 3.7\,$m/s.

b) Opposite direction:

$$\vec{p}_{tot,f} = \vec{p}_{tot,i}$$

$$m_{tot}\vec{v}_f = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$(60,000\,\mathrm{kg})\vec{v}_f = (40,000\,\mathrm{kg})(3.0\,\mathrm{m/s}\,\hat{x}) + (20,000\,\mathrm{kg})(-5.0\,\mathrm{m/s}\,\hat{x})$$

Then $\vec{v}_f = 0.33\,$m/s.