

Sample Test for Unit IV

Potentially useful equations:

$$F_s = -k(\Delta x) \quad |W_s| = \frac{1}{2}k(\Delta x)^2 \quad K = \frac{1}{2}mv^2 \quad P = \frac{W}{\Delta t}$$

$$\vec{p} = m\vec{v} \quad W_{\text{net}} = K_f - K_i \quad W_{\text{n.c.}} = E_f - E_i \quad \varepsilon = \frac{W_{\text{out}}}{W_{\text{in}}}$$

$$W = FD \cos \theta \quad W_{\text{cons.}} = U_f - U_i \quad \vec{F}_{\text{ave}}(\Delta t) = \Delta \vec{p} \quad E_{\text{tot}} = U_{\text{tot}} + K_{\text{tot}}$$

1. a.) Considering $W = FD \cos \theta$, the force and displacement are in the same direction, so $\theta = 0$ and $\cos 0^\circ = 1$. Then $W = (240\text{N})(5\text{m}) \cos 0^\circ = 1.2 \times 10^3$.

b.) $W = \vec{F} \odot \vec{D}$

$$W = [(100\hat{x} + 70\hat{y}) \text{ N}] \odot [(9.0\hat{x} - 2.0\hat{y}) \text{ m}]$$

$W = (900 - 140) \text{ J} = 760 \text{ J}$ (NOTE: There was an error in the answers on the handout! I had changed the question but changed 2b instead of 1b.)

2. How much work is done:

a.) The magnitude of the frictional force is $F_f = \mu F_N$, where from Newton's Law $F_N = mg$. So, $F_f = 86.24 \text{ N}$, and the work done is $W = F_f D \cos(180^\circ)$; $W = -3100 \text{ J}$. (The minus sign was left off from the handout. Sorry!)

b.) The spring constant comes from $F_{\text{app}} = k(\Delta x)$; $k = (70 \text{ N})/(0.02 \text{ m}) = 3500 \text{ N/m}$ (the wrong answer was updated. see 1b, Sorry!). Then the work done $W = \frac{1}{2}k(\Delta x)^2 = 2.8 \text{ J}$

3. a.) What is the Law of Conservation of Energy? Energy cannot be created or destroyed. The total energy of the universe (or any "closed" system) is constant. (The total energy can change in an "open" system smaller than the universe whereby energy enters or leaves the system.)

b.) $\varepsilon = \frac{W_{\text{out}}}{W_{\text{in}}}$ Here, $W_{\text{in}} = 180,000 \text{ J}$ and $W_{\text{out}} = 70,000 \text{ J}$, so $\varepsilon = 0.39$ (or 39%).

4. a) The ball rises by 4 cm from the release point to where it leaves the spring. By the work-energy theorem, $W_{\text{net}} = K_f - K_i$. The forces doing work are the spring (positive) and gravitational (negative) forces, and $K_i = 0$ (starts from rest), so

$$W_{\text{spring}} + W_g = K_f$$

$$\frac{1}{2}k(\Delta y)^2 - mg(\Delta y) = \frac{1}{2}mv_f^2$$

and solve for $v_f = 4.5 \text{ m/s}$. (Remember to convert $\Delta y = 4 \text{ cm}$ to 0.04 m .)

b) Note that at start and finish the velocity is zero, so $K_i = 0$ and $K_f = 0$, so

$$W_{\text{spring}} + W_g = 0$$

$$\frac{1}{2}k(\Delta y)^2 - mg(\Delta y) = 0$$

and solve for $\Delta y = 1.09 \text{ m}$.

5. The power is $P = W/(\Delta t)$. Since the velocity is constant, $W_{\text{net}} = 0$ (net force is zero or no change in kinetic energy), so the work done by the forklift is the work done against gravity (mgh). $P = (11760 \text{ J})/(5.0 \text{ s}) = 2352 \text{ W}$.
6. The average force can be found from the impulse: $\vec{F}_{\text{ave}}(\Delta t) = \vec{p}_f - \vec{p}_i$, where $\vec{p}_i = 3.5 \text{ kg m/s } \hat{x}$ and $\vec{p}_f = -7.0 \text{ kg m/s } \hat{x}$. Then with $\Delta t = 0.010 \text{ s}$, you get that $\vec{F}_{\text{ave}} = -1050 \text{ N}$. (Note that the negative sign indicates the arbitrary choice of direction as positive toward the batter and negative away from the batter.)
7. This is an “explosion” momentum problem. Momentum is conserved between just before and just after the shot because the net external force on the system is zero. The initial total system momentum (rifle plus bullet) is zero (at rest): $\vec{p}_{\text{tot},i} = 0$, so the total momentum just after the shot is also zero: $\vec{p}_{\text{tot},f} = 0$

$$\vec{p}_{\text{rifle},f} + \vec{p}_{\text{bullet},f} = 0$$

$$m_r \vec{v}_{r,f} + m_b \vec{v}_{b,f} = 0$$

Therefore,

$$\vec{v}_{r,f} = -\frac{m_b \vec{v}_{b,f}}{m_r}$$

If we take the bullet’s direction as the positive \hat{x} direction, this becomes

$$\vec{v}_{r,f} = -\frac{(0.010 \text{ kg})(900 \text{ m/s } \hat{x})}{2.0 \text{ kg}} = -4.5 \text{ m/s } \hat{x}$$

(The speed is then 4.5 m/s.)

8. Again, momentum is conserved between just before and just after the collision because the net external force is zero. So $\vec{p}_{\text{tot},f} = \vec{p}_{\text{tot},i}$.

$$\vec{p}_{\text{tot},i} = \vec{p}_{\text{car},i} + \vec{p}_{\text{truck},i}$$

$$\vec{p}_{\text{tot},i} = (3.6 \times 10^4 \text{ kg m/s}) \hat{x} + (3.0 \times 10^4 \text{ kg m/s}) \hat{x} = 6.6 \times 10^4 \text{ kg m/s}) \hat{x}$$

and

$$\vec{p}_{\text{tot},f} = \vec{p}_{\text{car},f} + \vec{p}_{\text{truck},f}$$

but the car and truck have the same velocity because they are stuck together, so

$$\vec{p}_{\text{tot},f} = (m_{\text{car}} + m_{\text{truck}}) \vec{v}_f$$

Then use $\vec{p}_{\text{tot},f} = \vec{p}_{\text{tot},i}$ and solve for \vec{v}_f ($= 14.7 \text{ m/s}$).