Physics 1114: Unit 5 Hand-out Homework (Answers)

Problem set 1

1. The flywheel on an experimental bus is rotating at 420 RPM (revolutions per minute). To find (a) the angular velocity in rad/s (radians/second), we need to convert revolutions to radians (2π radians per revolution) and minutes to seconds:

$$420\frac{\text{rev}}{\text{min}} \times \left(\frac{2\pi\text{rad}}{\text{rev}}\right) \times \left(\frac{\text{min}}{60\text{s}}\right) = 44\,\text{rad/s}$$

   (b) The linear (i.e., tangential) speed of a point 16 cm from the center of the flywheel is $v_t = r\omega = 7.04$ m/s.

2. A helicopter has blades that are 10 ft long extending from the axle. The tip of the blade should not exceed 1100 ft/s (the speed of sound). (a) The maximum angular velocity (ω) is

$$\omega = \frac{v_t}{r} = \frac{1100\,\text{ft/s}}{10\,\text{ft}} = 110\,\text{rad/s}$$

   (Note that arc length per sec divided by radius (length) gives radians per second, no need to convert to meters. The conversion won't hurt, but it cancels out because you would have the same factor in the numerator and denominator.) (b) 110 rad/s = 1050 RPM (invert the conversion from problem 1).

3. (a) A ball on the end of a string is whirled in a horizontal circle with 0.5 m radius at a rate of one revolution every two seconds (0.5 rev/s). The ball's centripetal acceleration (a_c) is $a_c = \frac{v^2}{r}$, so we find v from $v_t = r\omega$, but ω has to be rad/s, not rev/s: $\omega = 0.5\,\text{rev/s} \times (2\pi\text{rad/rev}) = 3.14\text{rad/s}$ (or π rad/s). Then $v_t = r\omega = (0.5\,\text{m})(3.14\,\text{rad/s}) = 1.57\,\text{m/s}$. So that

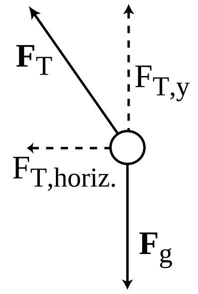
$$a_c = \frac{v^2}{r} = \frac{(1.57\,\text{m/s})^2}{0.5\,\text{m}} = 4.93\,\text{m/s}^2$$

   Alternatively, using $v_t = r\omega$,

$$a_c = \frac{v^2}{r} = \frac{(\mathrm{v_t}\omega)^2}{\mathrm{r}} = \mathrm{r}\omega^2$$

   and plug in for r and ω.

   (b) If the ball has a mass of 0.65 kg, The horizontal component of the tension in the string comes from Newton's 2nd Law in the centripetal (radial) direction: $\sum F_c = ma_c = mv^2/r$. The only force acting in the centripetal direction is the horizontal component of the tension because the only other force acting is the weight (gravity, $\vec{F}_g$, straight down):



So, $F_{\text{T,horiz}} = ma_c = (0.65\,\text{kg})(4.93\,\text{m/s}^2) = 3.2\,\text{N}$. The vertical component of the tension has to balance the weight ($a_y = 0$, so $F_{T,y} - F_g = 0$). Then $F_{T,y} = mg = (0.65\,\text{kg})(9.8\,\text{m/s}^2)$. Then combine the components of tension by the Pythagorean theorem and get $F_T = 7.13\,\text{N}$.

(c) Now the 0.65 kg ball is whirled in a vertical circle of 0.5 m radius. The maximum tension the string can withstand is 40 N. The maximum allowed speed of the ball is found from the maximum $a_c\,(= v^2/r)$ that the ball can get from the forces. The only forces acting on the ball are the tension and the weight.

i) At the top, tension and weight both act downward (the centripetal direction), so $F_T + F_g = mv_{max}^2/r$ ($v_{max} = 6.0\,\text{m/s}$).

ii) At the side (i.e., when the string is horizontal), only the tension acts in the centripetal direction, so $F_T = mv_{max}^2/r$ ($v_{max} = 5.5\,\text{m/s}$). and

iii) at the bottom of the circle, F_T again acts in the centripetal direction, but F_g acts in the *anti*-centripetal direction, so $F_T - F_g = mv_{max}^2/r$ ($v_{max} = 5.1\,\text{m/s}$).

4. A 1550 kg car is traveling at 12 m/s on a level road where the coefficient of static friction between the tires and the road is 0.80.

   (a) For a flat surface, it is the frictional force between the tires and the road that provides the net force to cause the change in direction (i.e., the centripetal component of the net force).

$$\sum F_c = ma_c$$
$$F_f = mv^2/r$$
$$\mu F_N = mv^2/r$$
$$\mu mg = mv^2/r$$

   Note that the mass divides out. Then solving for r:

$$r = \frac{v^2}{\mu g} = 18.4\,\text{m}$$

   (b) When it rains, the coefficient of friction on the road drops to 0.10. The maximum speed with which the car can safely negotiate a turn with the same radius (18.4 m) is found from the formula above: $v^2 = \mu g r$, or

$$v = \sqrt{\mu g r} = 4.25\,\text{m/s}$$

5. A wheel starts from rest ($\omega_i = 0$ rad/s) and rotates with constant angular acceleration (α). After $\Delta t = 6.0$ s have elapsed, it has rotated through $\Delta\theta = 25$ rad. (a) The angular acceleration comes from $\Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$

$$25\,\text{rad} = 0 + \frac{1}{2}\alpha(6.0\,\text{s})^2$$

   So that $\alpha = 1.39$ rad/s^2.

   (b) The angular velocity at t = 6.0 s can be found a couple ways. Since we found α, we could use $\omega_f = \omega_i + \alpha(\Delta t) = 8.33$ rad/s. Or, we could use $\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t$, solving for ω_f.

Problem set 2

1. A wheel on a moving car slows uniformly (α is constant and is negative) from 72 rad/s (ω_i) to 44 rad/s (ω_f) in 7.0 seconds (Δt). (a) The angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = -4\,\mathrm{rad/s^2}$$

(b) The angle through which the wheel turns in the 7.0 s interval is $\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t = 406\,\mathrm{rad}$. Or, using our value for α, we could use $\Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$.

2. A motorcycle wheel turning at 0.25 rad/s (ω_i) is brought to rest ($\omega_f = 0$) by the brakes in exactly two revolutions ($\Delta\theta = 2\,\mathrm{rev} = 4\pi\,\mathrm{rad}$). The angular acceleration of the wheel comes from $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$. Solve for α and plug in the numbers to get $\alpha = -2.5 \times 10^{-3}\mathrm{rad/s^2}$.

3. The sun has a mass of 1.99×10^{30} kg, while the mass of Mars is only 6.34×10^{23} kg. The radius of Mars's orbit is 2.28×10^8 km (2.28×10^{11} m). The force of attraction between these two bodies is

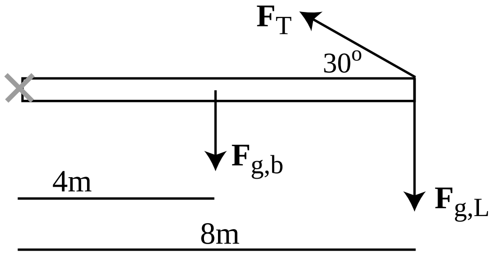
$$F_G = \frac{Gm_1m_2}{r_{1,2}^2}$$

$$F_G = \frac{\left(6.67 \times 10^{-11}\,\frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}\right)(1.99 \times 10^{30}\,\mathrm{kg})(6.34 \times 10^{23}\,\mathrm{kg})}{(2.28 \times 10^{11}\,\mathrm{m})^2}$$

$$F_G = 1.61 \times 10^{21}\,\mathrm{N}$$

4. The torque produced by a 70 lb force (311.5 N) applied to the end of a 9 inch (0.229 m) wrench and the angle between the force and wrench is 80°: $\tau = F_\perp r = Fr\sin\theta_{F,r} = 70.2\,\mathrm{N\,m}$

5. To find the tension in the cable, analyze the forces and torques. The forces include the tension (F_T), the weight of the load ($F_{g,L}$) and the weight of the beam ($F_{g,b}$). There are also forces at the end of the beam where it meets the wall, but we will put our torque reference point there so that their torques will be zero (moment arm of zero length). Now we use $\sum\vec{\tau} = I\vec{\alpha}$ and the equilibrium condition of $\vec{\alpha} = 0$. The two weights make torques (τ_b and τ_L) acting in a clockwise direction, so we'll make them positive, whereas the torque (τ_T) due to the tension acts in the counter-clockwise direction and will be negative:



$$\sum\vec{\tau} = I\vec{\alpha} = 0$$

$$\tau_b + \tau_L - \tau_T = 0$$
$$\tau_T = \tau_b + \tau_L$$

Now use $\tau = Fr\sin\theta_{F,r}$ for each:

$$F_T(8\,\mathrm{m})\sin 30^\circ = (20\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(4\,\mathrm{m})\sin 90^\circ + (40\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(8\,\mathrm{m})\sin 90^\circ$$

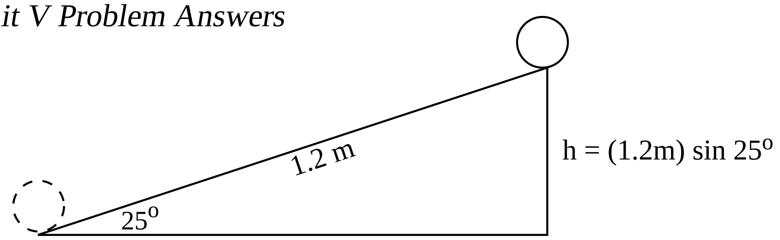
and solve for $F_T = 980\,\mathrm{N}$.

Problem set 3

1. As in problem 2-5, for the biceps muscle force problem we use $\sum\vec{\tau} = I\vec{\alpha}$ and the equilibrium condition of $\vec{\alpha} = 0$:
$$\sum\vec{\tau} = I\vec{\alpha} = 0$$
$$\tau_{\mathrm{arm}} + \tau_L - \tau_F = 0$$
$$\tau_F = \tau_{\mathrm{arm}} + \tau_L$$
$$F(0.05\,\mathrm{m})\sin 72^\circ = (9\,\mathrm{N})(0.14\,\mathrm{m})\sin 90^\circ + (28\,\mathrm{N})(0.33\,\mathrm{m})\sin 90^\circ$$
So that $F = 221\,\mathrm{N}$.

2. The moment of inertia is $I = 0.21\,ML^2$, where L is the arm length. To find the angular acceleration of an 8.0 kg arm, 0.66 m long, when subject to a (net) torque of $\tau = 27\,\mathrm{N\,m}$, we start with the relationship between net torque, moment of inertia, and angular acceleration (Newton's 2nd Law in torque form):
$$\tau_{net} = I\alpha$$
$$\alpha = \frac{\tau_{net}}{I}$$
$$\alpha = \frac{27\,\mathrm{N\,m}}{0.21(8.0\,\mathrm{kg})(0.66\,\mathrm{m})^2}$$
Then $\alpha = 37\,\mathrm{rad/s^2}$

3. A solid steel disk ($I = \frac{1}{2}MR^2$) has a radius of 0.52 m and a mass of 290 kg. To get an acceleration of 1.20 rad/s^2, we find the net torque (τ_{net}) as above:
$$\tau_{net} = I\alpha$$
$$\tau_{net} = \left[\frac{1}{2}(290\,\mathrm{kg})(0.52\,\mathrm{m})^2\right](1.20\,\mathrm{rad/s^2}) = 47\,\mathrm{N\,m}$$



4. A ball of mass 0.78 kg and radius of 0.06 m ($I = \frac{2}{5}MR^2$) starts from rest and rolls without slipping for a distance of 1.2 m down a 25° incline. We can determine the speed at that point by using the work-energy theorem: $W_{\text{net}} = K_{\text{tot,f}} - K_{\text{tot,i}}$, where K_{tot} is the sum of the linear and rotational kinetic energies. The ball starts from rest, so $K_{\text{tot,i}} = 0$:

$$W_{\text{net}} = K_{\text{tot,f}} - 0$$

The only work is done by gravity ($W_g = mgh$). The normal force is always perpendicular to the motion so it does zero work.

$$mgh = K_{\text{rot,f}} + K_{\text{lin,f}}$$

$$mgh = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2$$

$$mgh = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega_f^2 + \frac{1}{2}mv_f^2$$

Since the ball is rolling without slipping, $\omega = v/r$, so the radius r will cancel out after squaring. We can also divide out the mass:

$$gh = \frac{1}{5}v_f^2 + \frac{1}{2}v_f^2 = \frac{7}{10}v_f^2$$

Using $g = 9.8\,\text{m}\,\text{s}^2$ and $h = (1.2\,\text{m})\sin 25° = 0.507\,\text{m}$:

$$v_f = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10(9.8\,\text{m/s}^2)(0.507\,\text{m})}{7}} = 2.66\,\text{m/s}$$

Note that the final linear speed is *less* than for an object sliding without friction because part of the energy goes into rotation. If we write the moment of inertia as $I = xMR^2$, where x changes for different solid objects (spheres, disks, hoops), we see that v_f decreases as x increases. So the sphere's speed will be faster than a disk's speed for the same situation ($I_{\text{disk}} = \frac{1}{2}MR^2$) because $1/2 > 2/5$. And a hoop will be even slower ($I_{\text{hoop}} = MR^2$).

5. This is a conservation of (angular) momentum problem. The extra tension to pull the ball inward does not exert a external torque, so $\tau_{\text{net}} = 0$ and $L_f = L_i$. Using $L = I\omega$,

$$I_f\omega_f = I_i\omega_i$$

Where $I_i = mr_i^2$, $I_f = mr_f^2$, and $\omega_i = v_i/r_i = (2.2\,\text{m/s})/(1.2\,\text{m}) = 1.83\,\text{rad/s}$. Also, $r_i = 1.2\,\text{m}$ and $r_f = 0.7\,\text{m}$.

$$mr_f^2\omega_f = mr_i^2(1.83\,\text{rad/s})$$

The mass cancels out, and we solve for $\omega_f = 5.38\text{rad/s}$. So that $v_f = \omega_f r_f = 3.76\,\text{m/s}$. (The extra tension *does* do work, however, so K is not conserved!)