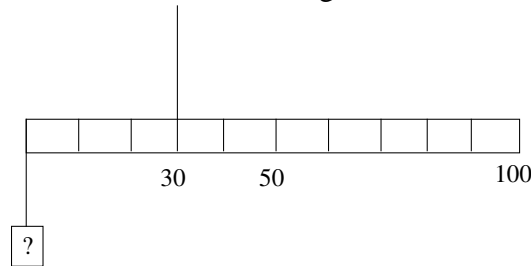


Sample Test for Unit V

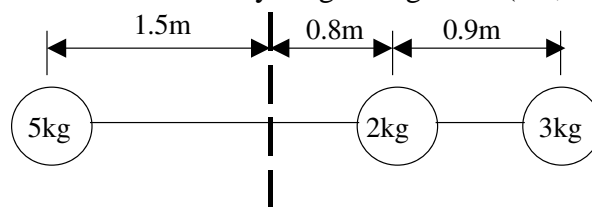
Potentially useful equations:

$$\begin{array}{llll} \omega_f = \omega_i + \alpha(\Delta t) & v_t = r\omega & K_{\text{rot}} = \frac{1}{2}I\omega^2 & \sum F_c = F_{\text{net},c} = ma_c \\ \Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2 & \vec{L} = I\vec{\omega} & W_{\text{net}} = K_{f,\text{tot}} - K_{i,\text{tot}} & \sum F_t = ma_t \\ \tau = F_{\perp}r \quad (= Fr \sin \theta) & \sum \vec{\tau} = I\vec{\alpha} & I = mr^2 \text{ (point mass)} & F_G = \frac{Gm_1m_2}{r_{1,2}^2} \\ \omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta) & a_c = \frac{v^2}{r} & K_{\text{lin}} = \frac{1}{2}mv^2 & \Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t \end{array}$$

- A wheel on a moving car speeds up uniformly from 10 rad/s to 46 rad/s in 12.0 seconds.
 - Find the angular acceleration of the wheel.
 - Find the angle (in radians) through which the wheel turns in the first 6.0 seconds.
- A bicycle is traveling at 15 m/s, if the wheels are 27 inches in diameter, what is their angular speed (in rad/s)? (1 inch = 2.54×10^{-2} m). (The wheels are rolling without slipping.)
- A centrifuge is used to separate particles of different sizes and densities suspended in liquids. A test tube is spun around at high speed so that the large acceleration causes the denser particles to migrate to the bottom of the test tube. If the bottom of the test tube is 9 cm from the axis of rotation, what rotation rate is needed to get a centripetal acceleration of 9800 m/s^2 ? (i.e., 1000 times g) First find the angular speed in rad/s and convert to rev/min.
- A uniform meter stick is suspended at the 30 cm mark by a string. The stick has a mass of 270 g, and its center of gravity is at the 50 cm mark. A counterweight hangs at the 0 cm mark to keep the stick balanced and horizontal. (a) What is the mass of the counterweight? (Hint, take your reference point to be the 30 cm mark so that the torque from the string becomes zero.) (b) What is the tension in the string?



- Three point masses are connected by a lightweight rod (i.e., neglect the mass of the rod).



- Find the moment of inertia relative to the rotation axis (heavy dashed line)
- If a 100 N net force is applied at 30° to the 5 kg mass, what will the angular acceleration be? (Find the torque first!)

6. Two point masses of 2 kg each are at opposite ends of a massless rod. The rod is 4 m long and is rotating about its center at an angular speed of 5 rad/s. By some internal mechanism the rod shrinks to half its original length. What is the new rotation rate, assuming momentum is conserved?
7. A solid wheel of mass 1.24 kg and radius 0.44 ($I = \frac{1}{2}MR^2$) starts from rest and rolls without slipping for a distance of 1.7 m down a 35° incline. How fast is it traveling at that point? (Note that with careful algebra, the mass and radius of the wheel divide out.)

Short Answers:

1. (a) $\alpha = 3 \text{ rad/s}^2$, (b) 336 rad.
2. 43.7 rad/s.
3. 330 rad/s = 3150 rev/min.
4. (a) $m = 180 \text{ g}$. Consider the mass of the stick to be concentrated at the 50 cm mark and balance the two torques. (b) By balancing forces ($F_{\text{net},y} = 0$), $F_T = 4.4 \text{ N}$.
5. (a) $I_{\text{tot}} = I_1 + I_2 + I_3$, where $I_1 = m_1 r_1^2$, etc. $I_{\text{tot}} = (11.25 + 1.28 + 8.67) \text{ kg m}^2 = 21.2 \text{ kg m}^2$ (Note that the right-most mass is 1.7 m from the axis!) (b) The torque is $\tau = (100 \text{ N})(1.5 \text{ m}) \sin 30^\circ = 75 \text{ N m}$, so $\alpha = 3.5 \text{ rad/s}^2$ (from $\tau = I\alpha$)
6. $I_i = (2 \text{ kg})(2 \text{ m})^2 + (2 \text{ kg})(2 \text{ m})^2 = 16 \text{ kg m}^2$, and likewise find that $I_f = 4 \text{ kg m}^2$. From conservation, $I_f \omega_f = I_i \omega_i$, so $\omega_f = 20 \text{ rad/s}$.
7. $v_f = 3.6 \text{ m/s}$