Answers to Sample Test for Unit V

Potentially useful equations:

$$\omega_f = \omega_i + \alpha(\Delta t) \qquad v_t = r\omega \qquad K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad \sum F_c = F_{\text{net},c} = ma_c$$

$$\Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2 \qquad \vec{L} = I\vec{\omega} \qquad W_{\text{net}} = K_{f,\text{tot}} - K_{i,\text{tot}} \qquad \sum F_t = ma_t$$

$$\tau = F_\perp r \ (= Fr\sin\theta) \qquad \sum\vec{\tau} = I\vec{\alpha} \qquad I = mr^2 \text{ (point mass)} \qquad F_G = \frac{Gm_1m_2}{r_{1,2}^2}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta) \qquad a_c = \frac{v^2}{r} \qquad K_{\text{lin}} = \frac{1}{2}mv^2 \qquad \Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$$

Longer Answers:

1. (a) Starting from $\omega_f = \omega_i + \alpha(\Delta t)$,

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$\alpha = 3\,\text{rad/s}^2$

(b) With $\Delta t = 6\,\text{s}$, use $\Delta\theta = \omega_i(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$ to find that $\Delta\theta = 114\,\text{rad}$.

2. When rolling without slipping, the linear velocity and angular velocity are related by $v_t = r\omega$. The radius of the wheel is

$$r = \frac{27\,\text{in}}{2} \times \frac{2.54 \times 10^{-2}\,\text{m}}{1\,\text{in}} = 0.343\,\text{m}$$

So $\omega = v_t/r = 43.7\,\text{rad/s}$ ($v_t = 15\,\text{m/s}$).

3. We need to have $a_c = 9800\,\text{m/s}^2$ and $r = 0.09\,\text{m}$, and we know that $a_c = v^2/r = \omega^2 r$ (from $v_t = r\omega$).

$$a_c = \omega^2 r$$
$$\omega = \sqrt{\frac{a_c}{r}}$$

$\omega = 330\,\text{rad/s}$ = 3150 rev/min

4. (a) Consider the mass of the stick to be concentrated at the 50 cm mark and balance the two torques: from the weight of the stick τstick (clockwise) and from the weight of the load τ_L (counter-clockwise). The torque equilibrium condition is $\sum\tau = 0$

$$\tau_{\text{stick}} - \tau_L = 0$$

$$\tau_{\text{stick}} = \tau_L$$

$$(0.27\,\text{kg})(9.8\,\text{m})(0.20\,\text{m})\sin 90^\circ = \text{m}(9.8\,\text{m})(0.30\,\text{m})\sin 90^\circ$$

m = 0.180 kg or 180 g

(b) By balancing forces ($F_{\text{net,y}} = 0$), $F_T = m_{stick}g + m_L g = 4.4\,\text{N}$.

5. (a) $I_{\rm tot} = I_1 + I_2 + I_3$, where $I_1 = m_1 r_1^2$, etc. $I_{\rm tot} = (11.25 + 1.28 + 8.67)\,{\rm kg\,m^2} = 21.2\,{\rm kg\,m^2}$ (Note that the right-most mass is 1.7 m from the axis!)

   (b) The torque is $\tau = (100\,{\rm N})(1.5\,{\rm m})\sin 30° = 75\,{\rm N\,m}$, so $\alpha = 3.5\,{\rm rad/s^2}$ (from $\tau = I\alpha$)

6. $I_i = (2\,{\rm kg})(2\,{\rm m})^2 + (2\,{\rm kg})(2\,{\rm m})^2 = 16\,{\rm kg\,m^2}$, and likewise find that $I_f = 4\,{\rm kg\,m^2}$. From conservation of momentum, $I_f\omega_f = I_i\omega_i$, and $\omega_i = 5\,{\rm m/s}$, so $\omega_f = 20\,{\rm rad/s}$.

7. See the answer to the handout problem (number 4 in set 3), which has identical procedure. The only real differences are the incline angle and length and the moment of inertia.

$$v_f = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.8\,{\rm m/s^2})(0.97\,{\rm m})}{3}} = 3.6\,{\rm m/s}$$