Accretion — capture of supercooled droplets by ice-phase precipitation particle
  If freeze immediately on contact → Rimed crystals or graupel
  If freezing is not immediate → hail

Aggregation — Clumping together of ice crystals to form snowflakes (R&Y); more generally, clumping together of ice phase particles
Particle fall speed is a fundamental quantity in the processes of accretion and aggregation.

Fall speeds are dependent upon habit and dimension.
Measured terminal velocities of ice crystals

![Graph showing terminal velocities of different types of ice crystals](image)

**FIG. 9.7.** Nakaya and Terada’s measured terminal velocities of ice crystals
(From Fletcher, 1962.)
Pristine habit fall speed \((-10 \, ^{\circ}\mathrm{C}; 1000 \, \mathrm{mb})\) \(v_t < 1 \, \mathrm{ms}^{-1}\)
Fall speed of aggregates ($v_t = 1-2 \text{ ms}^{-1}$)
Fall speed of solids (graupel, rimed crystals) \( (v_t = 1-3 \text{ ms}^{-1}) \)

Fig. 10-38. Best fit curves for fall velocity versus maximum dimension of single solid precipitation particles of various types. (From Locatelli and Hobbs, 1974; by courtesy of the authors; copyrighted by American Geophysical Union.)
Fall speed of small liquid drops ($r < 500 \mu m$) ($v_t = 1-5 \text{ ms}^{-1}$)

Variation with size of the terminal fall velocity of water drops smaller than...

(From Beard and Pruppacher, 1969; by courtesy of Amer. Meteor. Soc., authors.)
Fall speed of large liquid drops ($r > 500 \, \mu m$) ($v_t = 1-10 \, ms^{-1}$)

2. Variation with size of the terminal fall velocity of water drops larger than 500 $\mu$m (From Beard, 1976; by courtesy of Amer. Meteor. Soc., and the author.)
### Summary of hydrometeor fall speeds

#### C. Hydrometeors

Summary:

<table>
<thead>
<tr>
<th>species</th>
<th>diam.</th>
<th>density</th>
<th>shape</th>
<th>fall velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>cloud drop</td>
<td>2 - 100μm</td>
<td>1000(mks)</td>
<td>spherical</td>
<td>&lt;&lt;1m/s</td>
</tr>
<tr>
<td></td>
<td>(10μm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rain drop</td>
<td>100-5000μm</td>
<td>1000(mks)</td>
<td>oblate</td>
<td>4 - 8m/s</td>
</tr>
<tr>
<td></td>
<td>(1000μm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ice crystals</td>
<td>2μm-3mm</td>
<td>50-900(mks)</td>
<td>plate/column</td>
<td>&lt;&lt;1m/s to 1 m/s</td>
</tr>
<tr>
<td></td>
<td>(100μm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>snow aggregates</td>
<td>100μm-6mm</td>
<td>50-900(mks)</td>
<td>varied</td>
<td>.5 to 1.5m/s</td>
</tr>
<tr>
<td>graupel</td>
<td>&lt;5mm</td>
<td>200-600(mks)</td>
<td>spherical to</td>
<td>up to several m/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>conical</td>
<td></td>
</tr>
<tr>
<td>hail</td>
<td>&gt;5mm</td>
<td>800-900(mks)</td>
<td>spherical</td>
<td>up to several 10'sm/s</td>
</tr>
</tbody>
</table>
Summary of ice fall speeds

Slow → fast: pristine crystals → rimed droplets → graupel

Observations of particle fall speed are often fit to a relation like

\[ u = aD^b \]

Similar fits are used for mass-diameter relations, i.e.

\[ m = aD^b \]

R&Y Table 9.2 gives values of a and b for different habits. Compare these values of a and b to those for a perfect spherical ice particle.
Fall speeds of hydrometeors in a commonly used cloud model scheme (Lin et al. 1983)

- **Hail**
  - $\rho = 1.0 \text{ g/kg}$
  - $\rho = 0.9 \text{ g/kg}$
  - $\rho = 0.7 \text{ g/kg}$
  - $\rho = 0.5 \text{ g/kg}$

- **Rain**
- **Snow**

Velocity (m s$^{-1}$) vs. Mixing ratio (g kg$^{-1}$) for different densities of snow, rain, and hail.
Collection efficiency

• Accretion
  Aerodynamic problem
  “Sticking”

Ice crystals fall more slowly than liquid droplets. ∴ higher collision efficiencies (maybe)

*Collision efficiency* turns out to be very complex

Supercooled droplets most likely freeze on contact with ice phase particles. ∴ *coalescence efficiency* is probably unity
Growth by accretion and aggregation

Accretion rate is analogous to Eq. 8.15. Begin with 8.15, use chain rule, and note assumption used to obtain $dr/dm$:

$$\frac{dm}{dt} = \bar{E}M\pi r^2 u(r)$$

Aggregation (of snow and ice) is similar, but observe that difference in fall speed between snow and ice is nearly constant.

Represent ice crystal content $M$ [$\text{kg m}^{-3}$] as number density:

$$M = N\nu \rho_{ic}$$

$N$ is number density; $\nu$ is average crystal volume; $\rho_{ic}$ is crystal density
Growth by accretion and aggregation

Rework to be in terms of volume:

\[
\frac{dV}{dt} = \left(\frac{9\pi}{16}\right)^{1/3} EV^{2/3} Nv \Delta u
\]

Gives reasonable results from observations of graupel and snowflakes
Diffusive ice growth vs. liquid water coalescence

- Condensation (liquid) and diffusion (ice) alone can’t explain growth of mg-sized particles in realistic time scales.

- However, in cold clouds diffusive growth can produce ice particles that are large enough to precipitate, melt, and fall to the surface as drizzle.

- Condensation growth in warm clouds is far too slow to produce this same effect.

- Coalescence is REQUIRED to produce precip. in warm clouds.

- Compare diffusive ice growth to coalescence.

Assume:

Ice crystal — stellar dendrite; -15 °C; 800 mb; $10^{-8}$ g initial mass; supersaturated relative to water.

Droplet — 25 µm initial radius; 10 µm droplets w/ LWC = 1 g m$^{-3}$.
FIG. 9.8. Times required for an ice crystal and a water droplet (solid curves) to grow to the indicated mass. Top scale gives the corresponding drop radius. Dashed curves are for the rates of fractional increase of mass, referred to the scale on the right.
Diffusive ice growth vs. liquid water coalescence

Calculations show that a 4 µg particle (100 µm liquid) can be formed in:
   10 minutes by vapor deposition
   20 minutes by coalescence

Early (1950s) thought suggested that cold cloud processes were required because the 20 minute theoretical coalescence value was too slow to explain the observed onset of convective precipitation.

Since then, radar observations have shown that the first echo often appears at levels lower than the 0 °C line.

∴ Coalescence alone must explain initiation of precip. in warm clouds.

BUT, the discrepancy in timescale remains (theory is too slow compared to observations) and is still an active area of research to this day.
Radar observations relevant to aggregation (a preview)

- $Z \sim D^6$; for same geometric size, $Z(D_L) \approx 5 \times Z(D_i)$
- Above 0 °C level — ice crystals, aggregates, graupel, liquid
- Below 0 °C level — melting ice, rain

Radar reflectivity values are often a maximum near the freezing level

[adapted from Houze (1993)]

\[ V_R \approx \hat{V} \text{ if radar pointing vertically} \]
Radar observations (a preview)

1827 14 Dec. 1992
P-3 tail radar composite

Bright band
Radar observations (a preview)
Vertically-pointing mm-wave cloud radar
Bright band
What do raindrops look like?

Teardrops?

NO! Why is this a common misunderstanding?
What do raindrops really look like?

Hamburger buns!

- Bottoms are flat from aerodynamic effects of falling drops
- Smaller droplets are more spherical — smaller $v_t$
Rain

- Precip. often reaches the ground as rain
- Commonly measured (not necessarily very well)
- Does rain rate tell us much about the character of the precip?
- Drop size distribution (DSD) tells us MUCH more than rain rate

DSD is (#drops/unit volume)/unit size interval (e.g. \([\text{cm}^{-3} \mu\text{m}^{-1}]\)), such that if you integrate over all particle sizes you get the total concentration of droplets:

\[
N_{total} = \int_{0}^{\infty} N_d(D')dD'
\]

[Note: Typically wouldn’t be integrating to infinity, but since particles drastically fall off with size, it’s ok.]
DSD observations

1. Examples of measured drop-size distributions in rain. Indicated are the duration of the observation, the total number of drops and the average rainfall rate. Distributions 1 and 2 were recorded during steady constant rain; distribution 3 was recorded during a thunderstorm. (From Joss et al., 1968.)
FIG. 10.2. Measured drop-size distributions (dotted lines) compared with best-fit exponential curves (straight lines) and distributions reported by others (dashed lines). (From Marshall and Palmer, 1948.)
Drop size distributions

Typically —

- $N_d(D)$ is time-dependent and typically decreases with size
- High rain rates mean larger number of big drops
- DSD of precip. drops is approximately a negative exponential:

\[ N(D) = N_0 e^{-\Lambda D}, \]

where, as before, $N(D)dD$ is the drop concentration ($cm^{-3}$) between $D$ and $D+dD$.

Exponential form was first suggested by Marshall and Palmer (1948), based on observations from Ottawa, Canada during one summer.
Drop size distributions

Marshall and Palmer found that—

\[ N(D) = N_0 e^{-\Lambda D} \]

- \( \Lambda \) is a function of rain rate, which for their data was
  \[ \Lambda = 41R^{-0.21} \]
  \( \Lambda \) is \([\text{cm}^{-1}]\) and \( R \) is \([\text{mm h}^{-1}]\)

- Also, it turns out that \( N_0 = \text{const} = 0.08 \text{ cm}^{-4} \)

**WATCH OUT FOR UNITS — THEY WILL BITE**
Drop size distributions

• M-P isn’t perfect — not all DSDs have the simple exponential form
• Averaging many DSDs tends to produce an exponential curve
• For steady continental midlatitude rainfall, M-P is reasonable and used for radar estimation of rain rate (foreshadowing…)}