Simulated three-dimensional branched lightning in a numerical thunderstorm model

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[1] Lightning discharges are simulated by using a stochastic dielectric breakdown model within a numerical thunderstorm model with extensive parameterizations of electrification mechanisms. The lightning model simulates the macroscopic bidirectional extension of discharges as a step-by-step stochastic process. Discharge channels are propagated on a uniform grid, and the direction of propagation (including diagonals) for a particular step is chosen randomly, with the probability for choosing a particular direction depending on the net electric field. After each propagation step the electric fields are recomputed via Poisson’s equation to account for the effect of the conducting channel. The lightning parameterization produces realistic looking, three-dimensional, branched lightning discharges. A variety of lightning types have been produced, including intracloud discharges, negative cloud-to-ground (CG) lightning, and positive CG lightning. The model simulations support the hypothesis that negative CG flashes occur only when a region of positive charge exists below the main negative charge region. Similarly, simulated positive CG flashes were found to occur only in regions of storms where the two significant charge layers closest to ground had roughly a “normal dipole” structure (i.e., positive charge above negative).

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1. Introduction

[2] Lightning parameterizations are required for simulations of the electrical evolution of strongly electrified storms. Without lightning flashes the electric field can quickly reach magnitudes far greater than any that have been measured in actual storms. A number of lightning parameterizations have been presented in the literature, and each has some advantages as well as shortcomings. This paper presents a lightning parameterization derived from the dielectric breakdown model that was developed by Niemeyer et al. [1984] and Wiesmann and Zeller [1986] to simulate electric discharges through gases. Previous studies [e.g., Petrov and Petrova, 1993, 1999; Pasko et al., 2000] have also used the Wiesmann-Zeller breakdown model to study electrical discharges in the atmosphere, but they have been limited to two-dimensional slab-symmetric or axisymmetric domains and did not include cloud models. The present work extends the use of the Wiesmann-Zeller breakdown model in two ways. First, the model has been extended to a three-dimensional domain, which is required for the most realistic representation of electric field and thunderstorm dynamics. Second, this new lightning parameterization has been incorporated within a full simulation numerical cloud model, in which it has been able to produce a variety of lightning types and structures.

[3] Section 2 contains an overview of lightning parameterizations in the literature. Section 3 describes the Wiesmann-Zeller dielectric breakdown model and our adaptations to make it a lightning parameterization. In section 4, examples of simulated unipolar discharges are presented and qualitatively compared with laboratory discharges. Some examples of parameterized lightning flashes are also presented. We conclude with a description of some lightning behaviors observed in modeled storms.

2. Background

[4] Lightning parameterizations have fallen roughly into two categories: those creating “bulk” flashes and those with explicit lightning channels. An example of a bulk scheme is the parameterization of Rawlins [1982], which simply reduced the charge densities everywhere in the cloud model domain. Takahashi [1987] simulated lightning flashes by removing charge from the regions of highest charge density. Ziegler and MacGorman [1994] distributed charge to regions where the net charge density exceeded a threshold value. Such bulk schemes are relatively simple to implement but lack realism. Most parameterizations with explicit channels treat only a one-dimensional or unbranched channel. The branching of lightning channels would be extremely difficult to treat analytically because of the lack of symmetry and the incomplete understanding of the processes involved.

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The lightning parameterization of Helsdon et al. [1992] was the first to apply the concept of the bidirectional leader [Kasemir, 1960]. The parameterization could produce a single, unbranched channel that traced the electric field line from an initiation point. The channel was extended in both directions along the field line until the ambient field magnitude fell below a certain threshold at the locations of the channel endpoints. An idealized analytical model was used to calculate the positive and negative linear charge densities induced on the channel by the ambient electric field. The charge distribution at each end of the channel was adjusted to maintain net charge neutrality of the flash, consistent with the theory of Kasemir [1960]. The charge carried by the channel was deposited in the small ion category of the storm model, which explicitly treated ion capture by hydrometeors [Helsdon and Farley, 1987]. Flashes parameterized by Helsdon et al. [1992] were capable of reversing the sign of the net charge density at affected points. Two limitations of the Helsdon et al. parameterization were that cloud-to-ground (CG) flashes were not treated and development of the channel structure ignored the contribution to the electric field from the charge on the channel itself.

The independent parameterizations of Solomon and Baker [1996] and Mazur and Ruhnke [1998] produced only a single, vertical channel, but did treat the contribution to the electric field by the charge induced on the channel itself. Thus they used the net electric field rather than the ambient field to control propagation. (The net electric field is the sum of the ambient field plus the “local” field contribution from the conducting channel.) This is an important feature for a lightning parameterization, because the enhancement of the electric field at channel tips is what is thought to allow lightning propagation into regions of weak electric field.

Solomon and Baker [1996] used analytical equations to calculate the contribution to the electric field from the charge induced on a one-dimensional channel. Their parameterization produced both intracloud (IC) and negative cloud-to-ground (−CG) flashes when used with simple charge models [Solomon and Baker, 1996] or within a 1.5-dimensional cloud model [Solomon and Baker, 1998]. Mazur and Ruhnke [1998] assumed an axisymmetric “tripoole” charge arrangement for the storm (a main negative region with a main positive region above and a smaller axisymmetric “tripole” charge arrangement for the storm (a main developed. The electric field around interior branches (i.e., the “screening” lightning channels as the flash develops. The effects of the charge induced by the ambient field on the conducting channels are to enhance the electric field around channel extremities and to reduce the electric field around interior branches (i.e., the “screening” effect). A second important feature of the Wiesmann-Zeller model is that it produces branched channel structures.

P Petrov and Petrova [1993, 1999] used simple two-dimensional arrangements of charged conductors to produce the ambient electric field. Flashes began at the surface of a conductor, and channels propagated through the surrounding neutral regions. Petrov and Petrova [1999] also examined the effect of using a pressure-dependent threshold, instead of a constant threshold, for conventional breakdown. When the electric field threshold for propagation decreased with height, channels extended to higher altitudes than they did when the threshold was constant. However, the simple arrangement of charged volumes limited the applicability of the results to real thunderstorms.

Pasko et al. [2000] applied the Wiesmann-Zeller dielectric breakdown model to simulate the bidirectional development of sprite discharges high in the stratosphere. This model used an axisymmetric domain and produced realistic looking sprite structures. Discharges were initiated via a simple approximation of the effect of a CG flash on the electrostatic field at high altitudes.

The lightning model of Hager et al. [1989, 1998] also produced bidirectional, axisymmetric IC and −CG flashes. Hager’s model was completely deterministic, and channels were extended from all points where the electric field exceeded the breakdown threshold in any given step. The only substantive difference between Hager’s model and the breakdown model of Niemeyer et al. [1984] is the method used to solve Poisson’s equation and the process of choosing where to extend the channels. However, the deterministic growth process could be unrealistic: Garik et al. [1987] showed that for simulations of diffusion-limited aggregation (a similar process) a deterministic approach failed to simulate the proper geometry, which could be achieved only by having an element of random selection.

3. Electrical Discharge Models

The lightning parameterization presented here is derived from the dielectric breakdown model developed by Niemeyer et al. [1984] (hereinafter called NPW) and Wiesmann and Zeller [1986] (hereinafter called WZ). Dielectric breakdown models simulate only the macroscopic properties of discharge channels and do not deal with the microscopic processes of breakdown. Although the formulation is rather simple, the models produce complex structures that resemble actual electrical discharges. Our first step, described in section 3.1, was to use the unidirectional breakdown model to simulate the discharge experiments of Williams et al. [1985]. We then extended the model to parameterize bidirectional lightning in thunderstorms, as described in section 3.2.

3.1. Dielectric Breakdown Model

The NPW model was developed to examine the branched structures of dielectric breakdown. WZ made the NPW model more physical by adding a critical electric field threshold (Ecrit) for propagation and allowing for an internal electric field (Ein) in the discharge channel. The internal field simply specifies the voltage drop along the channel segment and accounts for the resistance of the channel. The most common application of the WZ model has been to simulate breakdown through insulating material between two conductors (e.g., flat plates or concentric spheres) to which a voltage difference is applied [e.g., Fowler et al., 1998]. Dissado and Sweeney [1993] formulated a different model specifically for solid dielectrics and achieved results similar to those of the WZ model. They simulated two-dimensional electrical channels that originated bidirectionally from a defect within the dielectric and were driven by an oscillating electric field.

The WZ model simulates the step-by-step propagation of a unipolar discharge from an initiation point by successively creating new breakdown extensions, or bonds, from the discharge structure. Figure 1 shows a sample two-dimensional grid with part of a discharge. Each new extension is chosen randomly from all possible new extensions. An extension from a grid point on the channel to an adjacent grid point is possible (i.e., has a nonzero
probability) if the electric field vector favors propagation toward the adjacent grid point and the electric field magnitude between the two points exceeds a critical value, $E_{\text{crit}}$. Once all possible extensions have been identified, the probability $p$ of choosing a particular extension is given by

$$p_i(E) = \begin{cases} \frac{1}{\pi} (E_i - E_{\text{crit}})^{\eta} & \text{for } E_i > E_{\text{crit}}, \\ 0 & \text{for } E_i \leq E_{\text{crit}}, \end{cases}$$

where $\eta$ is a weighting exponent and $E_i$ is the magnitude of the electric field component between the $i$th pair of channel and adjacent nonchannel points. The divisor $F$ is the sum of the unnormalized probabilities, $F = \sum_i (E_i - E_{\text{crit}})^{\eta}$, so $\sum p_i = 1$. Each probability is transformed into a range of width $p_i$ within the range $(0, 1)$:

$$p_i \rightarrow \left( \sum_{k=1,j=1}^{\eta} p_k \sum_{k=1,j=1}^{\eta} p_k \right)$$

and a random number selects one new extension.

[16] Most implementations of the WZ discharge model (including this one) add only one new extension at a time and then update the potential of the surrounding grid for the effect of extending the conducting channel. In reality, growth may occur simultaneously on different branches. For example, Femia et al. [1993] and Fowler et al. [1998] have attempted to deal with simultaneous growth.

[17] NPW had a critical field of zero ($E_{\text{crit}} = 0$) in their model and found that the density of branching decreases with increasing values of $\eta$. The outermost channel points (i.e., exposed channel tips) tend to have the largest electric field magnitudes (i.e., largest probabilities), while inner branches tend to have lower probabilities because the outer branches form a partial Faraday cage that reduces the electric field inside. Choosing $\eta > 1$ tends to increase the probabilities of possible extensions from the outermost points relative to those from interior points, so the interior becomes shielded more quickly. However, when $\eta < 1$ the probabilities for exterior and interior points are more nearly equal. Thus, with $E_{\text{crit}} = 0$, NPW found that “brush” type (densely packed) discharges result for $\eta \leq 1$, but “branched” structures result for $\eta \geq 3$.

[18] Wiesmann [1988] found that introducing $E_{\text{crit}} > 0$ (with $\eta = 1$) has a similar effect as setting $\eta > 1$, since both tend to limit side branching. Pietronero and Wiesmann [1988] argued that $\eta$ can be derived from the relationship between the electron avalanche velocity and the local electric field in gas discharges. However, Dissado and Sweeney [1993] pointed out the difficulty in physically justifying any exponent other than $\eta = 1$. We have therefore assumed a linear relationship between $p_i$ and $E_i$ (i.e., $\eta = 1$) as well as assuming $E_{\text{crit}} > 0$.

[19] In the WZ model, electrical resistance of the channels is represented by an internal electric field $E_{\text{int}}$. A perfect conductor would have $E_{\text{int}} = 0$, and its surface would have a constant potential, while a conductor with resistance has $E_{\text{int}} > 0$, and the potential varies along its length. A nonzero internal field acts to reduce the electric field between the channel and nonchannel points and thus reduces the overall extent of the discharge structure (all other values being held equal). When a new grid point is added to the discharge, the electric potential $\phi_{\text{new}}$ at the new channel point is calculated from the potential $\phi_0$ of the point from which the channel segment extends:

$$\phi_{\text{new}} = \phi_0 - s E_{\text{int}} d,$$

where $d$ is the length of the new segment and $s$ is the sign of charge carried by the channel. Thus the potential at a point that is $n$ segments away from the initiation point can be written in terms of the reference potential $\phi_{\text{ref}}$ of the starting point, so that (2) becomes

$$\phi(n) = \phi_{\text{ref}} - s \sum_{j=1}^{n} E_{\text{int}}(j) d_j,$$

where the sum is along the path from the initiation point to segment $n$. Each channel point has a unique path from the initiation point, because no branch is allowed to rejoin to another branch. Unipolar discharges are generally initiated from a ground plane, so that $\phi_{\text{ref}} = 0$. For bidirectional discharges (section 3.2) the reference potential is taken as the average of the ambient potentials of the two initial grid points.

[20] The electric potential $\phi$ at grid points not on the channel must be updated after each extension of the conducting channel. The potential is found by solving Poisson’s equation,

$$\nabla^2 \phi = -\frac{\rho}{\epsilon},$$

where $\rho$ is the net charge density at points not on the channel and $\epsilon$ is the electric permittivity. The electric potential is already defined by (3) along the discharge structure, which is treated as a boundary with Dirichlet conditions when solving (4). Because of the complex geometry of the discharge boundary, Poisson’s equation is most easily solved with an iterative technique such as successive overrelaxation (SOR), as described in Appendix A. The charge density induced on the channel at a grid point can be found simply from Poisson’s equation ($\rho = -\epsilon \nabla^2 \phi$). The charge on the channel is not needed for solving (4), because the effect of this charge on $\phi$ is included by satisfying the boundary condition imposed by the channel potential (equation (3)). For unipolar discharges the channel charge is not calculated until the discharge extinguishes (i.e., all growth probabilities become zero). We do calculate the channel charge periodically during development of bidirectional discharges, however, to check for overall flash neutrality (described in section 3.2.3).

[21] No timescale is assumed for the discharge development. Therefore currents are not calculated. The sequence in which channel segments are added can be considered a kind of time ordering, but it is somewhat problematic to assign an actual timescale to the simulated discharge. Part of the difficulty is that...
real discharges would be expected to have simultaneous extensions from different branches, but the present model does not accommodate simultaneous growth. For lightning simulations needing a timescale, one could estimate the elapsed time by using average measurements of positive and negative leader velocities [Marshall and Ruhnke, 1998], but this has not been attempted here.

3.2. Stochastic Lightning Model

The stochastic lightning model (SLM) is an application of the WZ model to simulate bidirectional discharges in regions of varying net charge density (e.g., in an electrified storm). The bidirectional model of lightning propagation is adopted here because of support from both theory and observation [e.g., Kasemir, 1960; Marshall and Ruhnke, 1993, 1998]. The main difference from the unidirectional WZ model is that the discharge channel may be extended from both ends (positive and negative) of an initial channel. This section describes the details of the SLM and the procedure for simulating lightning flashes in the context of a thunderstorm model.

When used within the electrified numerical thunderstorm model (section 4.2), the SLM propagates discharges in a domain whose overall size is the same as that of the thunderstorm model but whose grid spacing is the same in all three dimensions. The grid spacing for the SLM is kept less than or equal to the smallest spacing in the thunderstorm domain. The net charge density is interpolated linearly in three dimensions from the thunderstorm domain to the SLM grid. The equal spacing not only simplifies the numerics but also reduces the bias in the electric field calculations in different directions. For wide domains (80 × 80 km or larger) we find that 500-m spacing is the highest practical resolution while maintaining computational efficiency. Smaller domains (smaller storms) would allow better resolution of the lightning, but our focus has been on larger storms. Note that the stepped leaders photographed beneath clouds typically advance in segments on the order of 50 m, and tortuosity in lightning channels is also observed on even smaller scales. Although 500-m resolution obviously cannot simulate the branching and tortuosity observed on smaller scales, the larger-scale tortuosity and structure is simulated reasonably well, because of the fractal nature of lightning channels. Also, one of the main functions of the lightning parameterization is the redistribution of charge, and this is adequately performed.

Computationally, the channel diameter is the same as the grid spacing, an issue that we have not seen addressed in the literature. A channel diameter of 500 m is certainly not realistic, but it appears that the channel can be thought of as having a smaller diameter (and correspondingly higher charge density) within the larger grid. This idea was partially tested in the following manner. A flash was first propagated at 500-m resolution. The channel and the net ambient charge density were then interpolated onto a grid with doubled resolution (250-m spacing). This procedure is reasonable, because the coarser grid spacing, an issue that we have not seen addressed in the literature. Some recent observations indicate that lightning channels have at least a small resistance [Rakov et al., 1998]. Therefore the simulated lightning channels are assumed to have a nonzero internal electric field ($E_{\text{int}}$). The value of $E_{\text{int}}$ may be set to a constant value or to a fraction of the initiation threshold. The example flashes shown in section 3.2.3 all used constant values of $E_{\text{int}} = 500$ V m$^{-1}$.

3.2.1. Lightning initiation.

The decision process for initiating a lightning flash follows the procedure of MacGorman et al. [2001]. A flash occurs when the electric field magnitude exceeds the initiation threshold $E_{\text{init}}$ anywhere in the model domain. Our default choice for the initiation threshold is the height-varying “breakeven” or “runaway electron” electric field given by Marshall et al. [1995] [see also Gurevich et al., 1992],

$$E_{\text{init}}(z) = 167 \rho_a(z)$$

$$\rho_a(z) = 1.208 \exp(-z/8.4),$$

where $E_{\text{init}}$ has units of kilovolts per meter, and $\rho_a$ is the air density (in kilograms per cubic meter) as a function of altitude, $z$ (in kilometers).

In the present study, $E_{\text{init}}$ was modified to have a lower bound of 30 kV m$^{-1}$ and an upper limit of 125 kV m$^{-1}$. Marshall et al. [1995] found that the runaway electron threshold was a better fit than the conventional breakdown threshold to the maximum electric field magnitudes seen in many thunderstorm soundings. The actual processes of natural lightning initiation are not understood well. Whether or not the runaway electron hypothesis correctly describes lightning initiation, it at least provides a good match to the maximum electric field magnitudes observed as a function of height. We would expect similar height distributions of lightning initiation and structure if we used a pressure-dependent threshold for conventional breakdown, though the magnitudes of charge density and electric field in the storm would be affected.

Recent tests using a constant value of $E_{\text{init}}$ throughout the domain have also produced reasonable results.

Following MacGorman et al. [2001], the lightning initiation point is chosen randomly from among all the points where the electric field magnitude is greater than a lower threshold $0.9E_{\text{init}}$. The lower threshold and randomized choice are an attempt to allow lightning to occur over a larger range of locations, to account for some of the natural variability of lightning initiation due to unresolved subgrid-scale fluctuations in a cloud model. The discharge is started as a conducting leader channel between two grid points. The first point is chosen as described above, and the second is the adjacent grid point that is closest to the direction of the electric field vector. Positive and negative leaders are propagated from opposite ends of the initial channel. Positive leaders carry net positive charge and tend to propagate toward and through regions of lower electric potential and negative charge. Negative leaders do the opposite, carrying net negative charge and extending preferentially into and through regions of net positive charge and higher electric potential.

3.2.2. Channel characteristics.

Although the breakdown physics are different for negative and positive leader development, both types of leaders are treated the same way in the model. The positive and negative parts of the flash are propagated independently so that up to two new channel segments (one positive and one negative) may be added per step. Both ends have default initial propagation thresholds of $E_{\text{crit}} = 0.75 E_{\text{init}}(z)$. The factor of 0.75 was chosen based on qualitative comparisons between flashes propagated on test grids of 250- and 500-m grid spacing. On the 250-m grid a flash was made using $E_{\text{crit}} = E_{\text{init}}(z)$ and used as a control run. On the 500-m grid, electric field reductions were used until the flash attained nearly the same overall spatial extent as on the 250-m grid. This procedure is reasonable, because the coarser resolution does not represent gradients in the potential as well as the finer resolution, so the calculated electric field magnitudes tend to be smaller on the coarser grid.

Some recent observations indicate that lightning channels have at least a small resistance [Rakov et al., 1998]. Therefore the simulated lightning channels are assumed to have a nonzero internal electric field ($E_{\text{int}}$). The value of $E_{\text{int}}$ may be set to a constant value or to a fraction of the initiation threshold. The example flashes shown in section 3.2.3 all used constant values of $E_{\text{int}} = 500$ V m$^{-1}$.

3.2.3. Flash neutrality.

Following the theory of Kasemir [1960] and Mazur and Ruhnke [1998], we assume that the channel structure should maintain overall charge neutrality as long as neither end reaches ground. With their very simple storm charge distribution, Mazur and Ruhnke [1998] maintained charge neutrality by adjusting the electric potential of their vertical one-dimensional channel. For IC flashes the change in potential typically was small, but for CG flashes it was much larger. The amount of change in channel potential depends on the distribution of ambient potential, which of course depends on the storm charge distribution, so the charge distribution affects the characteristics of their simulated lightning.
[30] For computational simplicity our parameterization maintains near-neutrality (within 5%) by a technique other than adjusting the reference potential of the channel (although that method is also being tested). Instead, we parameterize the effect that adjusting the reference potential would have on the growth of the lightning structure. In our channel propagation scheme we assume that the main effect of changing the potential would be to alter the electric field surrounding the channel, thereby altering the probabilities used to extend the flash. To mimic this effect, therefore, we adjust the electric field threshold for propagation at each end of the flash and also allow the end with less charge to add extra points, if possible. Of course, if a flash becomes a CG, then neutrality is no longer assumed.

[31] In our scheme for enforcing rough neutrality, the charge carried by positive and negative channel segments is computed periodically (e.g., after every fifth propagation step) by using Poisson’s equation (equation (4)). The magnitude of charge on positive segments is then compared with that on negative segments. If the magnitude of one polarity is more than 5% larger than that of the other, the electric field threshold for propagation is increased for that polarity, thereby retarding subsequent growth of the corresponding end of the flash. Likewise, growth is promoted for channels of the polarity having less charge by reducing the propagation threshold for that polarity. Additionally, growth is temporarily stopped for the channels with greater charge so that the opposite end can “catch up.” If the difference in charge subsequently falls below 5%, then the propagation thresholds are changed in increments back toward the default values. Adjustments are made in increments of 0.05 $E_{\text{init}}$($z$), and the minimum allowed threshold is 0.3 $E_{\text{init}}$($z$). These values were chosen arbitrarily and were assumed to be reasonable given the lack of knowledge about the propagation of actual lightning channels.) No maximum limiting value is specified. If growth cannot be forced, even at the minimum threshold, the flash is simply halted. Otherwise, the flash develops until all probabilities for growth fall to zero.

3.2.4. Treatment of CG flashes. [32] Until one of the leaders reaches ground, every flash is treated as a cloud flash in all respects. Once a leader reaches ground, however, propagation is stopped for all the branches of the same polarity as the one that reached ground. A negative leader reaching ground results in a $-$CG flash, which effectively lowers negative charge to ground from the storm. Similarly, a positive leader connecting to ground makes a positive cloud-to-ground (+CG) flash. Once a leader reaches ground, however, propagation is stopped for all the branches of the same polarity as the one that connected to ground and the loss of charge from the model domain. The upward growing (i.e., opposite polarity) leaders, however, are allowed to continue propagating until they stop naturally (or reach a domain boundary). This continues growth of the upward growing portion of a CG flash is not intended to mimic individual strokes or interstroke processes but to simulate the combined upward structure of all processes in a CG flash.

[34] The classification of a flash as a CG has some complications. In the initial tests of the lightning parameterization within the thunderstorm model, it was noticed that a branch often would extend downward below 3 km altitude without continuing all the way to ground. A number of shortcomings may cause this apparent failure to make a CG flash. For example, the coarse resolution of the channel grid results in significant underprediction of the electric field at channel tips. This underprediction has greater consequences at lower altitudes, where $E_{\text{crit}}$ (and therefore $E_{\text{init}}$) is larger. Also, the charge balancing procedure may underestimate the effect of the changing channel potential for developing CG flashes. Thus, for simplicity, a flash is presently classified as a CG when a leader branch reaches down to 1.75 km altitude (AGL). We will continue to examine this issue, especially in light of observations indicating that IC flashes can occur at relatively low altitude [Proctor, 1991; Koshak and Krider, 1989; Murphy et al., 1996]. Although IC/CG ratios are obviously affected by this uncertainty in identifying CG flashes, the total lightning flash rate and dominant CG polarity are not.

3.2.5. Charge redistribution. [35] The redistribution of lightning charge begins with the transfer of the charge from the fine grid to the coarser grid. Then the lightning charge field is smoothed to avoid sharp gradients in the final charge distributions. Since the thunderstorm model does not explicitly treat ion attachment processes, the ions released by lightning are assumed to be instantaneously distributed directly to the hydrometeor categories. Each hydrometeor category receives charge in proportion to its total surface area, regardless of preexisting charge. As given by Ziegler and MacGorman [1994]:

$$\delta \rho_i = \frac{\sigma_i}{\sum_k \sigma_k} \delta \rho,$$

where $\delta \rho_i$ is the charge density at a grid point on the lightning channel and $\delta \rho$ is the charge density deposited on the $i$th hydrometeor category having a total surface area $\sigma_i$. If branches extend outside the cloud to regions with no hydrometeors, the charge on branches outside the cloud is considered lost from the storm and is not tracked by the model.

4. Simulated Discharges

4.1. Unipolar Discharges

[36] An initial test of the WZ model was to simulate unipolar discharges produced in the laboratory by Williams et al. [1985]. Williams et al. exposed plastic slabs to an electron beam, thereby creating two-dimensional horizontal charge distributions in the plastic. They found that the discharges became more extensive as the charge density in the plastic increased (Figure 2).

[37] Figures 3 and 4 show simulations of discharges into slabs with horizontally uniform charge. The only difference between the two is that in Figure 3 the channels have zero internal electric field (i.e., the channels are treated as perfect conductors) and in Figure 4...
they have a small internal field (i.e., the channels have nonzero resistance). Figure 4 compares well qualitatively with the laboratory discharges (Figure 2). As the charge density was increased, the maximum propagation distance and degree of branching increased in both the experiment and the simulation. Both also exhibit a narrowing structure with increasing distance from the initiation point.

[38] Williams et al. [1985] also presented discharges in slabs that had small regions of one charge density embedded in a region of different charge density (Figure 6 of Williams et al. [1985], reproduced here in Figure 5). One experiment had pockets of higher charge density surrounded by a region of lower charge density, and a second trial had the same pattern but with switched charge densities (i.e., lower density charge pockets surrounded by a region of higher density charge). They observed preferential propagation and denser branching of the discharge in the regions of larger charge density. The corresponding simulations with enhanced or depleted regions showed very similar behavior (Figure 6).

[39] The simulations in Figures 3, 4, and 6 were scaled to atmospheric dimensions. The grid spacing was 125 m in all three dimensions. The charge densities were within the range of values determined from electric field and particle charge measurements in thunderstorms. The charge regions were confined to a layer that was 7 grid points in thickness. The central layer had the given charge density and was 3 grid points in thickness. The charge density decreased linearly above and below the central layer to zero at the top and bottom of the layer. This distribution approximates the assumed gaussian distribution of charge in the laboratory experiments [Williams et al., 1985]. The simulated discharges were initiated at a “needle” conductor extended from the ground plane. The propagation threshold was set at 100 kV m$^{-1}$.

[40] One might ask the question, how comparable are discharges in plastic slabs to lightning discharges? There is no doubt that they are different. It probably is reasonable to expect that some general behaviors are similar, such as faster and more branched propagation in regions of higher charge density [Williams et al., 1985]. It appears less likely to us that the internal field for a small laboratory discharge would be comparable to that of a lightning leader. Certainly there is no one value of internal field valid for the complete development of any particular discharge, and the choice for our parameterization is necessarily a very rough estimate.

4.2. Example Simulated Lightning Flashes

[41] The lightning model has been integrated into a full cloud simulation model, which was used to produce examples of simulated lightning for this paper. The numerical cloud model [Straka and Anderson, 1993] is three-dimensional and includes detailed bulk microphysics, with separate categories for cloud water, rain, cloud ice (columns, plates, and rimed), snow aggregates, frozen drops, three graupel densities, and two size ranges of hail. Details of the model dynamics are given by Carpenter et al. [1998]. Electrification parameterizations include noninductive (independent of the ambient electric field) and inductive (field-dependent) charge separation. The examples shown here used a noninductive scheme based on the laboratory work of Saunders and Peck [1998] for charge separation by rebounding collisions between ice crystals and actively riming
precipitation ice particles. Three other laboratory-based non-inductive collisional charging parameterizations are also optionally available, based on Takahashi [1978], Ziegler et al. [1991], or Brooks et al. [1997]. Inductive charging is allowed in rebounding collisions of graupel or hail with cloud droplets as shown by Ziegler et al. [1991]. The model also includes a parameterization for electrical screening layer development [Ziegler et al., 1991]. It should be noted that the screening layers at cloud boundaries are poorly resolved, with an assumed thickness of 500 m.

The storm simulations that produced the results presented here used atmospheric soundings that produced multicell or supercell storms. The multicell storm simulation used the analytical thermodynamic and shear profiles of Weisman and Klemp [1982, 1984], and a half-circle hodograph with arc length \( U_0 \) of 10 m s\(^{-1}\). The supercell storm used a composite of two atmospheric soundings from the Texas panhandle on 2 June 1995 [Gilmore, 2000]. Further details of the cloud model and environments are given by Mansell [2000].

Two simulated intracloud flashes are shown in Figures 7 and 8. The intracloud flash in Figure 7 is from the multicell storm ~40 min after the first lightning flash. The second IC flash (Figure 8) is from the supercell storm (~30 min after it produced

**Figure 6.** Simulations of unidirectional discharges with regions of enhanced or reduced charge density in which (a) small square regions have \( \rho = -1.2 \, \text{nC m}^{-3} \) surrounded by \( \rho = -0.6 \, \text{nC m}^{-3} \) and (b) charge densities are exchanged. Discharges were propagated in a three-dimensional domain and had an internal electric field of 1 kV m\(^{-1}\). Initiation point, at the top edge of the lower left small square in each plot, is from a needle protrusion from the simulated ground plane. Area shown is 100 by 100 grid points (12.5 × 12.5 km).

**Figure 7.** An intracloud (IC) flash from the multicell storm simulation at 3795 s, viewed from the side and from above. Initiation point at 8.25 km altitude is indicated by the diamond between the positive (shaded) and negative (black) branches. Magnitude of the dipole moment of the flash was 173 C km, and the flash deposited 36.3 C of charge at each end. Axes show distance in kilometers.
its first lightning flash). Both IC flashes exhibit two-layer ("bilevel") horizontal structure associated with two regions of charge in the storm: an upper positive charge region and a lower negative region (the "main positive" and "main negative" charge regions). Similar lightning structure has been observed by several investigators, including MacGorman et al. [1981] and Shao and Krehbiel [1996].

Both positive and negative CG flashes have been produced by the storm simulations. An example of a −CG flash from the multicell storm simulation is shown in Figure 9. The flash began at an altitude of 6.75 km, between the main negative region and a lower positive charge region near the base of the updraft. The downward propagating negative leader has noticeable horizontal extent in this example, but the amount of horizontal branching varies from flash to flash. The upward positive leaders have branching that is nearly exclusively horizontal, as in the simulated IC flashes.

Figure 10 shows an example of a positive CG flash (+CG) from the supercell storm simulation. The positive leader (shaded) propagated through the main negative charge region to ground. The negative leaders (black) branched extensively into the main positive charge region. Flash lowered 82.3 C to ground. Axis labels show distance in kilometers.

5. Discussion

Lightning flashes in the model appear always to originate between regions of opposite charge, as expected, where the electric
field reaches the largest magnitudes (Figure 11). This is true not only for IC and −CG flashes (Figures 11a and 11b), consistent with observations, but also for +CG flashes (Figures 11c and 11d). The initiation locations are always found at or near the boundary between positive and negative charge. Some initiation points appear farther away from the charge boundaries in Figure 11, because the slices show lightning activity in a slab that is 7 km (7 grid points) wide, and the charge contours are taken at the center of the slab.

Another notable effect of the simulated lightning is the occasional polarity reversal produced by lightning in the storm’s net charge density. This effect is evident in Figure 11b where a pocket of negative charge appears within the main positive region (e.g., at \( z = 11 \) km and \( x = 43 \) km) and positive charge pockets appear in the negative region (e.g., at \( z = 7 \) km and \( x = 39 \) km). The net charge polarity reversals can be caused by single flashes or by the cumulative effect of repeated flashes. Similar reversals were produced by the lightning parameterizations of Helsdon et al. [1992] and Solomon and Baker [1998].

An interesting model result is that IC lightning flashes often are initiated in the same region as CG flashes within a few minutes of each other or less (e.g., Figures 8 and 10). One must ask whether this is natural behavior or an artifact of our model. Observations indicate that IC and CG lightning can initiate from similar altitudes. Proctor [1991] reported that the height distribution of lightning initiations for South African storms was bimodal. The upper group of initiations was composed almost exclusively of IC flashes, but the lower group had both CG and IC flashes. Koshak and Krider [1989] and Murphy et al. [1996] found that for Florida thunderstorms, IC flashes occasionally occurred low in the storms, and they hypothesized that these IC flashes were discharges between a lower positive charge region and the main negative charge region tapped by −CG flashes. What remains to be tested is whether our parameterization produces the correct proportion of CG flashes and low IC

Figure 11. Lightning composites (left) and contour slices (right) for (a, b) simulated multicell storm with −CG flash (c, d) and supercell storm with +CG flash. Composites (Figures 11a and 11c) are surfaces over the regions of positive (blue) and negative (red) lightning leaders during a 2.5-min period. Green surfaces indicate initiation point locations. Slices (Figures 11b and 11d) again show lightning activity regions within 3 km of the charge contour plane (red for negative leaders, blue for positive leaders, and green for initiation locations) overlaid with charge density contours (solid for positive and dashed for negative). Contour values start at ±0.1 nC m\(^{-3}\) with intervals of 0.3. Cloud boundaries (droplets and ice crystals) are indicated by gray surfaces (left) or thick gray line (right). See color version of this figure at back of this issue.
flashes, though relatively few observational studies are available for this comparison.

[40] In our model, −CG flashes always began between the main negative charge and lower positive charge (Figure 11b), as inferred by Clarence and Malan [1957]. The modeling study of Solomon and Baker [1998] also found that −CG flashes occurred only after the development of a sufficiently strong lower positive charge region. Similarly, +CG flashes also always began between opposite charges in our simulations and occurred only where the lowest significant charge region near the initiation point was negative. This result for −CG flashes is consistent with the finding by Takeuti et al. [1978] and Brook et al. [1982] that winter thunderstorms in Japan that produced +CG flashes had a normal dipole structure (positive charge above negative). More recently, Carey and Rutledge [1998] inferred from corona point sensor measurements that +CG flashes emanated from regions of a storm where the lowest significant charge region was negative.

[50] The model results concerning +CG flashes offer a possible clarification of the conclusions of Takeuti et al. [1978] and Brook et al. [1982]. Takeuti et al. inferred from surface electric field measurements that the gross charge structure of +CG-producing thunderstorms had a positive charge region above and displaced horizontally from a lower negative charge region (the “tilted dipole”). A common interpretation of this charge configuration is that the key to +CG flashes is the “unshielding” of positive charge (i.e., the removal of the negative charge from beneath the positive charge) so that the positive charge would become “exposed to ground.” The simulation results strongly suggest that even if this unshielded charge arrangement were to occur, a negative charge region would still be needed to initiate +CG lightning. Another important factor may be the altitude of the positive charge region involved in +CG flashes. Brook et al. [1982] found that the source charge heights for +CG flashes tended to be only slightly greater than heights for −CG flashes. The present thunderstorm model offers the opportunity to further investigate the circumstances of +CG flashes.

6. Conclusions

[51] The lightning parameterization presented here, an extension of the Wiesmann-Zeller model of dielectric breakdown, has been found to be a useful tool in the study of lightning characteristics in numerical thunderstorm models. The successful simulation of laboratory discharges and the realistic structure and location of simulated lightning flashes lend credence to the model and to comparisons with lightning behavior in observed storms.

[52] The stochastic lightning model can produce a wide variety of lightning structures, including both CG flashes and bilevel IC flashes often observed by lightning mapping systems [e.g., Shao and Krehbiel, 1996; MacGorman et al., 1981; Rison et al., 1989]. Simulated flashes also reproduced observed features such as cloud-to-air discharges in which part of the flash extends beyond the cloud boundary. Brook et al. [1985] reported on photographs showing lightning channels extending above thunderstorms, particularly in the vicinity of overshooting tops. In our simulations both positive and negative channels may extend above the cloud near the overshooting top.

[53] The model has also produced all major types of lightning, including IC, −CG, and +CG flashes. The type of flash and the polarity of CG lightning depend on the charge distribution, which in turn depends on storm type, history, and other factors. The −CG flashes produced by the model are consistent with the hypothesis that a lower positive charge region is critical for their development [e.g., Clarence and Malan, 1957; Williams, 1989]. This hypothesis has been in the literature for many years and is widely accepted. One of the results of our storm simulations is that a similar hypothesis should apply to positive CG flashes: a negative charge region is needed below a positive charge region to promote positive CG flashes.

[54] The results reported here support the idea of using lightning mapping data to infer some of the gross features of a thunderstorm’s charge distribution. For example, simulated negative leaders tend to propagate through positive net charge, and positive leaders tend to propagate through negative net charge, as hypothesized by MacGorman et al. [1981], Williams et al. [1985], and Shao and Krehbiel [1996]. Although the region inhabited by a given polarity of leaders differs significantly from the region inhabited by the opposite polarity of thunderstorm charge (especially in the vicinity of recent lightning), the two correspond well enough in our simulations that regions filled with leaders are expected to give a rough idea of the location of net charge. This relationship and the tendency for lightning to begin on or near the boundary between oppositely charged regions can be used to provide considerable information about thunderstorm charge.

[55] Having three-dimensional lightning mapping data with other storm data from a wide variety of storm types also is essential for our ongoing modeling research. Present lightning mapping technologies [e.g., Krehbiel et al., 2000] offer ample opportunities for our present comparisons with model results. Such data enable us both to test various aspects of our cloud model and lightning parameterization and to investigate the storm processes responsible for observed lightning characteristics. Though interpreting storm model results requires an accounting for the limitations of even the most sophisticated models, simulations of observed storms are required to examine, in a self-consistent way, how the various individual electrical behaviors predicted by theory or observed in the laboratory affect the electrical properties of the very complex system that is a thunderstorm.

Appendix A: SLM Procedure

[56] Before a discharge begins the electric potential is calculated from Poisson’s equation (equation 4) for the model domain using the FISHPACK package (developed by J. Adams, P. Swartztrauber, and R. Sweet), which applies fast Fourier transforms in the horizontal directions and solves the resulting tridiagonal system of equations in the vertical direction. The bottom of the domain (ground) is set at zero potential (Dirichlet boundary condition), and the top and sides have Neumann condition ∂Φ/∂n = 0. The potential is solved using extended lateral boundaries to increase the accuracy of the solution by reducing the influence of the mirror charges that necessarily result from the boundary condition. A future improvement will be to extend the top boundary as well.

[57] Iterative methods are commonly used to solve the Poisson equation (equation 4). In our lightning parameterization the standard technique of successive overrelaxation (SOR) is used to update the solution in a subdomain around the developing lightning flash. The unidirectional version of our parameterization used the same SOR methods, but the solution was updated throughout the entire domain. After each channel extension, one or more SOR iterations are required to recompute the electric potential, using the solution from the previous extension as a first guess. At the beginning of each relaxation sweep through the grid, the SOR routine calculates the residual r (the estimated error in the solution) at each grid point by using

\[ r = (\Delta x)^2 (\nabla^2 \phi^* + \rho / \epsilon), \]  

(A1)

where \( \Delta x \) is the uniform grid spacing, \( \phi^* \) is the solution to the potential from the most recent \( n \)th iteration, and \( \rho \) and \( \epsilon \) are the
net charge density and electric permittivity, respectively. The Laplacian ($\nabla^2 \phi$) is calculated by a standard seven-point finite difference formulation (second-order accuracy), which is the
sum of the second derivatives in each direction:

$$\nabla^2 \phi = \sum_{i=1}^{\nu} \frac{\partial^2 \phi}{\partial x_i^2},$$

(A2)

where

$$\frac{\partial^2 \phi}{\partial x_i^2} \approx \phi_{(x_i - \Delta x_i)} - 2 \phi_{(x_i)} + \phi_{(x_i + \Delta x_i)} \Delta x_i^2.$$  

(A3)

Each SOR sweep updates the solution to the potential from the residual by

$$\phi_{jk}^{n+1} = \phi_{jk}^{n} + \frac{1}{\omega} \rho_{jk},$$

(A4)

where $\omega$ is the overrelaxation parameter and has an experimentally determined value in the range 1 to 2. After updating, $r$ is calculated at each grid point, and the maximum fractional residual $\varepsilon = \max(r)/\max(\phi)$ is determined. Relaxation iterations continue until $\varepsilon \leq 5 \times 10^{-3}$.

An increase in computational efficiency was achieved by adjusting the domain in which the Poisson equation is updated during the development of a lightning flash. When a new channel segment is added to the discharge structure, the electric potential will be significantly affected at the nearby points (assuming a 1/r dependence). It was found to be sufficient to recalculate the potential by the SOR method in a relatively small subdomain around the new channel point. After every ten propagation steps, relaxation is also performed in a subdomain that encompasses the entire lightning flash. This “local/global” method is effective in solving for the important short wavelengths (i.e., the high wave number Fourier components of the solution to the potential). The method is similar to, though it was developed independently of, the one employed by San±udo et al. [1995]. The long wavelengths are not as relevant to the process since the solution at points far from the flash is not important to the propagation of the discharge. The local solution is performed in a subdomain that is 21 grid points (10 km) wide in each direction and is centered on the new point. The global solution is calculated in a subdomain that contains the entire flash and is increased in size by two grid points on each side after each SOR sweep. The subdomain boundaries usually use Dirichlet conditions. However, if the boundary of a subdomain is within 10 grid points of a model domain boundary, then the smaller subdomain is expanded all the way to that boundary and uses the corresponding condition for the model domain (Neumann condition for top and sides and Dirichlet condition for bottom). This strategy of using a limited subdomain does not appear to introduce any obvious instabilities to the solution. Only rarely has a parameterized flash shown behavior that seems pathological (e.g., horizontal extension well beyond the cloud boundary).

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References


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